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CERTAIN PECULIARITIES OF TRANSPORT PHENOMENA IN SUSPENSIONS WITH INTERNAL ROTATIONS

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The concept of internal rotations in ferrosuspensions is shown to make possible the explanation of a number of hydrodynamic phenomena in magnetic fluids in the presence of rotating magnetic fields, and the prediction of qualitative peculiarities of mass transfer and of electrical conductivity of suspensions with rotating particles. It is shown that the convective mass transfer near rotating particles results in the relative increase of the effective coefficient of diffusion in suspension in proportion to the square of the Péclet number determined by particle parameters. The possibility of appearance in suspensions of coherently rotating particles of an effect similar to that of the Righi —Leduc of heat conduction in a magnetic field is indicated. The part played by convective transport in suspensions of coherently rotating parts is also illustrated by the examination of electrical conductivity of suspensions of spherical particles in a weakly conducting dielectric medium.

Hydrodynamic phenomena in magnetizable and polarizable media in the presence of rotating fields and of internal rotations induced by these were investigated in [1-9]. It was shown in [1-4] that the rotation of particles in a rotating field, due to the finite time of magnetization or polarization relaxation, generates a macroscopic motion of the medium only in the presence of moments of stresses induced by the diffusion of the internal angular moment. Experiments had shown [5] that, contrary to opinions expressed in [6], the macroscopic motion of medium with rotating particles occurs only in the presence of three-dimensional inhomogeneities. Motions of magnetizable suspensions in a rotating magnetic field in the presence of moving boundaries was investigated theoretically in [7, 8] and experimentally in [5, 9]. The effect of particle rotation the improvement of the effective coefficient of diffusion in the motion of blood was estimated in [10].

1. The effective coefficient of heat and mass transfer may be considerably increased in suspensions with rotating particles. Below, the effective coefficient of diffusion is determined for a suspension of spherical particles of radius a coherently rotating at the angular velocity $\omega = (0, 0, \omega)$. The convective transfer is, unlike in [10], considered with allowance for three-dimensional velocity distribution near rotating particles. Particle surface is assumed to be impermeable to the diffusing substance. The effective coefficient of diffusion is determined by averaging over cells.

Let at some distance from a particle the concentration distribution be defined by $c = c_0 + G_x x' + G_y y' + G_z z'$, and the equation of mass transfer be of the form

$$(\mathbf{v}\nabla) c = D \triangle c, \quad \mathbf{v} = a^3 [\omega \times \mathbf{r}'] \mathbf{r}'^{-3} \tag{1.1}$$

The solution of Eq. (1.1) with boundary conditions

$$\left.\frac{\partial c}{\partial r'}\right|_{r'=a} = 0, \quad c\,|_{r'\to\infty} = c_0 + G_i x_i'$$

is obtained in the form

$$c = c_0 + R_1 \cos\vartheta G_z + \operatorname{Re}(R \sin\vartheta\, e^{i\varphi})\, G_x + \operatorname{Im}(R \sin\vartheta e^{i\varphi})\, G_y$$

where ϑ and φ are angles of the spherical coordinate system whose polar axis lies along the z' -axis. After separation of variables, for functions R and R_1 we obtain equations

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \left(\frac{2}{r^2} + \frac{iP}{r^3}\right)R = 0, \quad P = \frac{a^2\omega}{D} \tag{1.2}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_1}{dr}\right) - \frac{2R_1}{r^2} = 0 \tag{1.3}$$

where $r = r'a^{-1}$ is the dimensionless distance from the particle center and P is the Péclet number.

By the introduction of new variables u and t in conformity with formulas $R = r^{-1/2} u$, $t = r^{-1/2}$ Eq. (1.2) is reduced to the equation of modified Bessel functions

$$\frac{d^2u}{dt^2} + \frac{1}{t}\frac{du}{dt} - \left(\frac{9}{t^2} + 4iP\right)u = 0$$

As the result, we obtain a solution of Eq. (1.2) of the form

$$R = r^{-1/2}\,[A_1 I_3(\lambda r^{-1/2}) + A_2 K_3(\lambda r^{-1/2})], \quad \lambda = 2\sqrt{iP} \tag{1.4}$$

where I_3 and K_3 are modified Bessel functions of the first and second kind and constants A_1 and A_2 are determined by the boundary conditions

$$A_1 = -A_2\frac{K_3(\lambda) + \lambda K_3'(\lambda)}{I_3(\lambda) + \lambda I_3'(\lambda)}, \quad A_2 = a(\lambda/2)^3$$

The corresponding solution of Eq. (1.3) is of the form

$$R_1 = a\,(r + {}^1/_2 r^{-2}) \tag{1.5}$$

As the result of averaging the obtained solutions, we obtain formulas for averaged quantities which determine the behaviour of suspension in the macroscopic scale. As the averaging volume we take a sphere of radius l, with $2l$ equal to the mean distance between suspension particles. For the mean concentration gradient over the cell

$$\langle\nabla c\rangle = V^{-1}\int \nabla c\, dV = V^{-1}\oint c\mathbf{n}\, dS$$

using (1.4) and (1.5) we obtain the formulas

$$\langle\nabla c\rangle_{x,y} \approx G_{x,y}\,(1 - {}^1/_2 P^2\varphi^{2/3}) \mp G_{y,x} P \varphi^{1/3} \tag{1.6}$$
$$\langle\nabla c\rangle_z = G_z\,(1 + {}^1/_2\varphi)$$

accurate to terms of second order of smallness with respect to $P\varphi^{1/3}$ $P \gg 1$, and $((al^{-1})^3 = \varphi$, where φ is the portion of particles by volume.

For high $P\varphi^{1/3}$ the effect of particles on one another is significant, and an extension of boundary conditions to infinity becomes impossible. The averaging of the total mass flow $\mathbf{J} = -D\nabla c + \mathbf{v}c$ is effected according to formula

$$\langle J_k \rangle = V^{-1}\int J_k dV = V^{-1}\int \frac{\partial}{\partial x_i}(x_k J_i)\,dV$$

Taking into consideration (1.4) and (1.5), and that at the cell surface $J_n = -Da^{-1}\partial c/\partial r$, for the averaged flow $\mathbf{J}$ we obtain

$$\langle J_{x,y}\rangle = -DG_{x,y}\,(1 + {}^1/_2P^2\varphi^{2/3}) \qquad (1.7)$$

$$\langle J_z\rangle = -DG_z\,(1-\varphi)$$

When $\langle\nabla c\rangle_x = 0$, taking into account (1.6) and (1.7) we have

$$\langle J_x\rangle = -D\cdot{}^1/_2P\varphi^{1/3}\,\langle\nabla c\rangle_y \qquad (1.8)$$

$$\langle J_y\rangle = -D\,(1 + {}^3/_4P^2\varphi^{2/3})\langle\nabla c\rangle_y$$

$$\langle J_z\rangle = -D\,(1 - {}^3/_2\,\varphi)\langle\nabla c\rangle_z$$

Formulas (1.8) show that in a suspension of coherently rotating particles an effect is present which is similar to the Righi-Leduc effect of heat conduction in a magnetic field and consists of the appearance of a flow in the direction normal to the macroscopic concentration gradient and to the angular rotation velocity of particles. The obtained form of dependence of the effective diffusion coefficient in the second of formulas (1.8) corresponds to that given in [10]. The third of formulas (1.8) shows that in the direction of particle rotation the effective diffusion coefficient corresponds to the known formula for that coefficient in the case of suspension of impenetrable particles (*).

The transfer phenomenon in a heterogeneous system is, according to [11], determined by the flow averaged over the surface. In this case

$$\int_\Sigma J_n dS = \Sigma n_k \langle J_k\rangle$$

where Σ is an arbitrary section of a spherical cell by a plane, $\mathbf{n}$ is a normal to that section, and $\langle\mathbf{J}\rangle$ is the flow averaged over the volume in conformity with the indicated above procedure.

2. The indicated peculiarities of the transfer phenomenon in suspensions or rotating particles may also occur in the field of electric charge transfer. Let us consider a suspension of spherical particles of radius a in a medium whose permittivity and conductivity are ε_1 and γ_1, respectively. The permittivity and conductivity of the solid phase are, respectively, ε_2 and γ_2. We examine the effective coefficient of electrical conductivity of such suspension on the assumption that particles ro-

*) Martseniuk, M.A., Thermal conductivity of ellipsoidal ferromagnetic particles in a magnetic field. 8-th Conference on Magnetohydrodynamics, Riga. Report Theses, Vol. 1. Riga, "Zinatne", 1975.

tate at angular velocity ω, and the electrical Reynolds number, which represents the ratio of convective and conductive transfer of an electric charge [12], is fairly high.

If $\mathbf{E}_0$ defines the homogeneous electric field at infinite distance from a particle, the potential of the electric field near that particle is determined by the Laplace equation with boundary conditions at the particle surface [12] and at infinity

$$\begin{aligned}
&D_{n1} - D_{n2} = 4\pi\sigma, \quad \psi_1 = \psi_2, \quad \psi|_{r\to\infty} = -(\mathbf{E}_0\mathbf{r}) \\
&\mathbf{D} = \varepsilon\mathbf{E}, \quad \mathbf{E} = -\nabla\psi, \quad \mathbf{i} = \gamma\mathbf{E}, \quad \mathbf{j} = \sigma\,[\boldsymbol{\omega}\times\mathbf{r}] \\
&-(R\sin\vartheta)^{-1}\frac{\partial}{\partial\vartheta}(j_\vartheta \sin\vartheta) - (R\sin\vartheta)^{-1}\frac{\partial j_\varphi}{\partial\varphi} = i_n^{(1)} - i_n^{(2)} \\
&(\boldsymbol{\omega} = (\omega, 0, 0), \quad \mathbf{E}_0 = (0, E_y, E_z))
\end{aligned} \tag{2.1}$$

The solution of the problem is of the form

$$\begin{aligned}
&\psi_1 = -(\mathbf{E}_0\mathbf{r}) + Aa^3(\mathbf{E}_0\mathbf{r})r^{-3} + Ba^3\mathbf{E}_0\,[\mathbf{e}_x\times\mathbf{r}]\,r^{-3}, \quad r \geqslant a \\
&\psi_2 = C(\mathbf{E}_0\mathbf{r}) + B\mathbf{E}_0\,[\mathbf{e}_x\times\mathbf{r}], \quad \mathbf{r} \leqslant a \\
&A = \frac{\gamma_2^\circ - \gamma_1^\circ + (\varepsilon_2^\circ - \varepsilon_1^\circ)(\omega\tau)^2}{1+(\omega\tau)^2} \\
&B = 3\frac{(\gamma_1^0\varepsilon_2^0 - \gamma_2^0\varepsilon_1^0)\,\omega\tau}{1+(\omega\tau)^2}, \quad C = -3\frac{\gamma_1^\circ + \varepsilon_1^\circ(\omega\tau)^2}{1+(\omega\tau)^2} \\
&\tau = \frac{1}{4\pi}\frac{2\varepsilon_1+\varepsilon_2}{2\gamma_1+\gamma_2} \\
&\varepsilon_k^\circ = \frac{\varepsilon_k}{2\varepsilon_1+\varepsilon_2}, \quad \gamma_k^\circ = \frac{\gamma_k}{2\gamma_1+\gamma_2}, \quad k = 1, 2
\end{aligned} \tag{2.2}$$

where τ is the charge relaxation time and $\omega\tau$ has the meaning of the electrical Reynolds number [12].

The effective coefficient of electrical conductivity of suspension is obtained by averaging the conduction current over the volume containing many particles, and the convection current $\sigma\mathbf{v}$ averages over the surface of spherical inclusions contained in the considered volume. The total averaged current in the suspension with both mechanisms of electric charge transfer taken into account may be defined as

$$\langle \mathbf{I}\rangle_k = -\gamma_1\langle\nabla\psi\rangle_k + V^{-1}\gamma_1\int_{\Sigma S_i}\psi n_k dS + V^{-1}\int_{\Sigma S_i} x_k i_n^{(1)} dS \tag{2.3}$$

$$\left(\langle\mathbf{I}\rangle = V^{-1}\int \mathbf{i}dV + V^{-1}\int_{\Sigma S_i}\mathbf{j}dS, \quad \langle\nabla\psi\rangle = V^{-1}\int\nabla\psi dV\right)$$

where $\langle\nabla\psi\rangle$ is the electric potential gradient averaged over the volume. On the assumption of equality of surface and volume averages it can be calculated for a heterogeneous system by averaging over a plane whose characteristic dimension is considerably greater than the mean distance between particles. In the case of diluted system when the perturbations of the field by particles can be considered as additive, with allowance for (2.2) we obtain

$$\int \nabla\psi \mathbf{n} dS = -E_{0z}(S-\Sigma) + E_{0z}\Sigma(2A+C) - 3BE_{0y}\Sigma$$
$$\Sigma = \sum_i \pi(a^2 - z_i^2), \quad \mathbf{n} = (0, 0, 1)$$

where $\mathbf{n}$ is a normal in the averaging plane, z_i is the distance of particle center to the plane, and summation is carried out over the totality of particles whose centers lie at distance $|z_i| \leqslant a$ from the averaging plane. Taking into account formulas (2.2) and that for uniform distribution of particles $\Sigma = S\varphi$, from the last equality we have

$$\int \nabla\psi \mathbf{n} dS = -E_{0z} S - 3S\varphi BE_{0y} + 3S\varphi AE_{0z}, \qquad \mathbf{n} = (0, 0, 1) \quad (2.4)$$

Similarly

$$\int \nabla\psi \mathbf{n} dS = -E_{0y}S + 3\varphi SAE_{0y} + 3\varphi SBE_{0z} \quad (2.5)$$
$$\mathbf{n} = (0, 1, 0)$$
$$\int \nabla\psi \mathbf{n} dS = 0, \qquad \mathbf{n} = (1, 0, 0) \quad (2.6)$$

For the electric current averaged over a plane of area S and normal $\mathbf{n} = (0, 0, 1)$

$$S\langle I_z\rangle = \int i_n dS + \oint_{\Sigma l_i} \sigma \mathbf{v}_n dl$$

(summation in the second term is carried out over the totality of contours of intersections of the plane with spherical particles) on the assumption of additiveness of perturbations of the electrical potential contributed by individual particles, we obtain

$$S\langle I_z\rangle = \gamma_1 E_{0z}(S-\Sigma) - (\gamma_1 A + \gamma_2 C - \omega\tau(2\gamma_1+\gamma_2)B) \times$$
$$\Sigma E_{0z} + (2\gamma_1+\gamma_2)(1+\omega\tau(A+(\varepsilon_1^\circ - \varepsilon_2^\circ)))E_{0y}\Sigma$$

where the last expression yields $S\langle I_z\rangle = \gamma_1 SE_{0z}$.

Similarly by averaging over a plane with normal $\mathbf{n} = (0, 1, 0)$ we obtain $S\langle I_y\rangle = \gamma_1 SE_{0y}$. Using formulas (2.4) and (2.5) and assuming the quality of gradients of volume and surface averages with $\langle\nabla\psi\rangle_y = 0$, we obtain

$$E_{0y} = -\langle\nabla\psi\rangle_z 3\varphi B, \quad E_{0z} = -\langle\nabla\psi\rangle_z(1+3\varphi A)$$

accurate to first order terms with respect to the volume portion of particles, and, respectively, for components of the surface-averaged current we have

$$\langle I_z\rangle = -\gamma_1\langle\nabla\psi\rangle_z(1+3\varphi A), \quad \langle I_y\rangle = -\gamma_1\langle\nabla\psi\rangle_z 3\varphi B$$

which, as can be readily shown, is the same as the volume averaging by formula (2.3). The current averaged over an arbitrary oriented plane with the normal $\mathbf{n}$ owing to the equation $\operatorname{div}\mathbf{i} = 0$ in solid and liquid phases, and boundary condition (2.1) is determined in accordance with the relation

$$S\langle I_n\rangle = \int i_n dS + \int_{\Sigma l_i} j_n dl = n_y\langle I_y\rangle + n_z\langle I_z\rangle$$

It follows from the derived laws of electrical conductivity of suspensions of rotating particles that for the effective coefficient of electrical conductivity the following formula is valid:

$$\gamma_e = \gamma_1 (1 + 3\varphi A)$$

The transfer of electric charge in similar media is, moreover, associated with the effect that is similar to the Righi —Leduc effect, i.e. with the presence of electric current that is transverse to the macroscopic electric field and to the angular velocity of rotating particles.

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